TMA4275 Lifetime Analysis (Spring 2020)
Exercise 6

Problem 1 – The two-parameter exponential distribution

The two-parameter exponential distribution has density

$$f(t;\theta,\gamma) = \frac{1}{\theta}\exp\left\{-\frac{t-\gamma}{\theta}\right\} \text{ for } t \geq \gamma$$

where $\theta > 0$, $\gamma \geq 0$.

Assume that we have a right censored sample (y_i, δ_i), $i = 1, \ldots, n$ from this distribution. Assume that all censorings take place at or after time γ.

a) Find the log-likelihood function $l(\theta, \gamma)$ for these data.

b) Let $(\hat{\theta}, \hat{\gamma})$ be the maximum likelihood estimators of (θ, γ). Why do we have

$$\hat{\gamma} \leq y_{(1)}$$

where $y_{(1)}$ is the smallest time among $y_1, \ldots, y_n$?

Then find explicit expressions for $(\hat{\theta}, \hat{\gamma})$. Show in particular that we always have $\hat{\gamma} = y_{(1)}$

c) In the lectures we have considered a likelihood method for constructing confidence intervals for one of two parameters in a model. The method uses the following:

Let $\hat{\gamma}(\theta)$ be the MLE of γ when θ is given. Then

$$W(\theta) = 2(\ell(\hat{\theta}, \hat{\gamma}) - \ell(\theta, \hat{\gamma}(\theta)))$$

is approximately χ^2_1 when θ is the true parameter. (Note that $\tilde{\ell}(\theta) = \ell(\theta, \hat{\gamma}(\theta))$ is the so-called profile log likelihood of θ).

Explain how this can be used to construct a confidence interval for θ. Do the calculations of the interval as far as you get.

d) Use MINITAB to estimate the parameters when the Pike cancer data (see page 25 of Slides 10 from lectures) are assumed to follow a two-parameter exponential distribution.

e) Reconsider the assumption in the beginning of the Problem, that all censorings take place at or after time γ. Can you think of cases where censorings also before time γ are possible? In such cases, how should the analysis in b) be modified?

Problem 2 – Censoring and truncation

$n = 10$ units with exponentially distributed life times and MTTF= θ are put on test. At time $c = 10$ the test is ended (type I censoring), and $r = 4$ units have failed by that time. The observed lifetimes are

$$0.9,\ 2.8,\ 5.9,\ 7.4$$

a) Write down the likelihood function and compute the MLE for θ. Which are the assumptions behind this approach?

b) Assume now that at the end of the experiment ($c = 10$) one does not know how many units were put on test, but only knows that the experiment has gone for 10 time units, with $r = 4$ failures at the times given.

How can you write down a likelihood for this case? (Hint: This is right truncation, see page 3 of Slides 11 from lectures).

Which are the assumptions behind this likelihood?

c) Maximize the likelihood in (b) to find the MLE under the conditions given there.

Problem 3 – Weibull regression

Do Problem 1 b,c in Exam 2013V.