Failure modeling and maintenance optimization for a railway line

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Abstract

This paper presents a model for deterioration and repair of a railway line. The critical failure is “broken line”. Two main failure mechanisms are considered: either shock failure, i.e. an immediate critical failure (without “warning”), or the critical failure occurs as the result of a degradation process, i.e. a degraded failure (crack) occurs first. Various types of inspection and maintenance are performed on the line. Inspection by Ultrasonic Inspection Cars (UIC) is carried out at regular intervals, and there is a probability q that the inspection shall detect a degraded failure. Additional inspection will be initiated on a segment of the line if degradation above a certain level is observed. A piece of rail which is degraded is more prone to suffer a critical failure (broken line), and when the degradation has reached a certain level, this will require immediate repair. The degradation/repair process within the fixed inspection interval is modeled as a time continuous Markov chain. Also the change of state implemented at the end of an inspection interval is modeled as a (time discrete) Markov chain. The model is based on actual inspection and failure data for a specific railway line in Norway. These data are used to estimate the parameters of the model. The given failure/maintenance model and estimation technique should generally be useful for systems that experience deterioration and are subject to imperfect inspection.

Key Words – Maintenance modeling, Markov chains, Phase type distributions, Inspection

1. Introduction

This paper is concerned with the modeling and statistical analysis of the failure mechanism of a railway line. The background is a study of failure and inspection data from *Dovrebanen*, which is a part of the Norwegian railway line from Trondheim to Oslo. A failure model for railway lines is presented, using a Markov model, and the transition rates are estimated from these data.

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The critical failures (broken rail) can either be seen as shocks (i.e. with no "warning"), or as a gradual degradation, where the line goes through various degraded states (with cracks) until it gets a critical failure. When a degraded failure occurs, the railway line is still functioning, and the crack can only be revealed by inspections of the line. Those inspections are performed at regular intervals by Ultrasonic Inspection Cars (UIC). However, at each inspection there is only a probability q of detecting a degraded failure; where q is roughly estimated to be between 0.4 and 0.7. A piece of rail which is degraded is more prone to suffer a critical failure than a piece of rail not degraded (i.e. in the OK state). When a critical failure occurs, the failure has to be repaired in order to maintain regular traffic.

The model will demonstrate for instance how various reliability parameters depend on the inspection interval, and will thus support identification of the most cost effective preventive maintenance strategy for the railway line in question. Other lines may have different conditions, and line specific data with a new estimation of parameters will then be required. However, the presented failure/maintenance model and estimation technique is believed to be useful also for other applications involving imperfect inspections and a gradual development of failure.

A preliminary model and results were reported in [1], and this was based on work by Dolven [2]. Both this and the current paper depend heavily on [3] and [4]. The same railway application was also studien in [5], mainly using a simulation approach. Further, the approach is based on standard Markov Chain theory, e.g. see [6], and the use of phase type distributions, see e.g. [7].

2. The failure and Maintenance Model

The overall model is based on two Markov models, one time continuous model describing the development between two inspections, and one Markov chain describing the transitions that may occur at an inspection, [3]. We start by describing the failure model.

2.1 Failure Model

A phase type distribution (see e.g.) is used for time to failure. The failure model includes two different states for degraded failures and two different states for critical failures. In addition we have the OK state (see Figure 1).

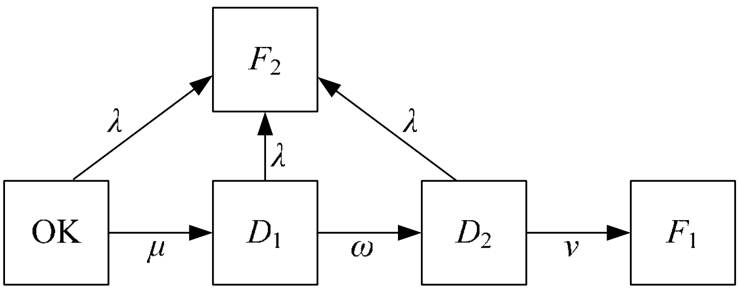

Figure 1: The Markov failure model

The critical failures can be divided into two categories; failures due to degradation, denoted F_1 and "shock" failures, denoted F_2. The latter failures happen when the rail is exposed to large external forces like rolling stock. Those failures cannot be avoided by inspection. The critical failures due to degradation, however, can be avoided by performing preventive repair if they are discovered at inspections.

The first degraded state, denoted D_1, is for minor degraded failures (cracks). If a rail is detected in this state, the observations are made more frequent so that a critical failure due to degradation not should be possible. When the degraded state called D_2 is detected (larger cracks) the failure is repaired immediately.

The development of degraded and critical failures is modeled by a time continuous Markov chain, see Figure 1. If the railway line does not have a critical failure, there is a constant rate λ for getting a shock failure F_2. In order to reach the critical failure state F_1 the rail has to go through the degraded states D_1 and D_2.

We partition the rail into small pieces of rail so that one piece is in only one of the states OK, D_1, D_2, F_1 or F_2, (and the above rates refer to these small pieces of length 1 m).

2.2 Time Continuous Markov Chain

In the following illustrations we will for simplicity ignore the state F_2. The effect of this failure category can afterwards be incorporated by just adding an additional failure rate, λ (cf. Figure 1).

However, in order to model also the maintenance for degradation failures we split the degradation states, according to whether these are detected or not. A subscript *u* on the degraded states indicates that a degraded failure is undetected (by UIC). Likewise, a subscript *d* indicates a degraded failure that is detected. Thus,

D_{1u} = Minor degraded failure (crack) being undetected.
D_{1d} = Minor degraded failure detected by UIC; then the observations are made more intensive (frequent) so that a critical failure due to degradation is not possible.
D_{2u} = Major degraded failure (crack) being undetected; (state believed to be OK).
D_{2d} =Undetected major degraded failure when the piece of line earlier has been detected to be in state D_1; and is therefore it is closely monitored, but it is *not* known that the state D_2 is reached. As soon as the state D_2 is detected the failure is repaired immediately and the piece of rail goes to OK.

If we start in one of the states OK, D_{1u} or D_{2u} at the beginning of an inspection interval, we get a time continuous Markov chain as illustrated in Figure 2. This will be valid for the complete inspection interval of length, T. However, if we during the inspection detect that the line is in the state D_1, then the next inspection interval will start in state D_{1d}, and then the more general diagram of Figure 3 applies. Here ρ refers to the transition rate caused by additional inspection in a detected state D_1 as explained above. The direct rate to OK follows by the assumption of immediate repair. For simplicity we introduce an absorbing state OK*, and transfer to OK is carried out at the start of the next inspection interval. So both the states F_1 and OK* are made absorbing states in the time continuous chain, meaning that a fresh start in OK always takes place at the beginning of an interval. This means that the same piece of rail can never have two failures or visits to D_{2u} within the same interval, and we do not *start* in OK in the middle of an interval. This is a computationally simplifying assumption that will not to a great extent affect our results.

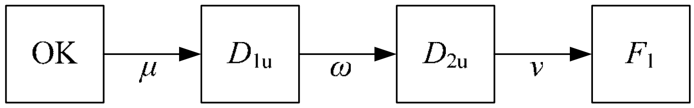

Figure 2: Time continuous Markov chain: start in OK (or D_{1u}, D_{2u}) at the beginning of an inspection interval

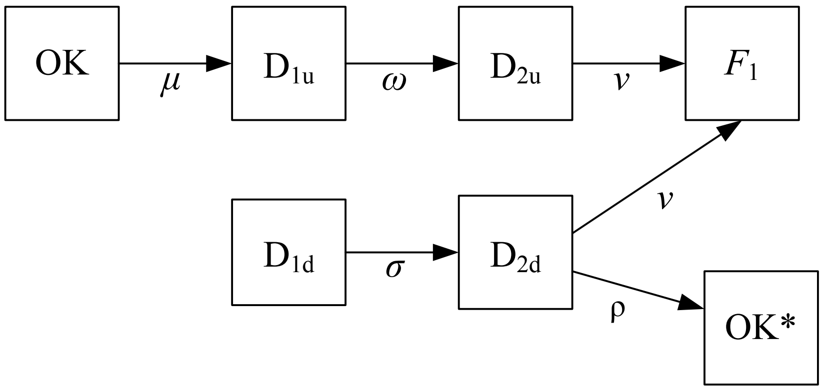

Figure 3: Time continuous Markov chain

Further, note that the modeling allows transitions from D_1 to D_2 to have different rates, depending on whether the degradation to D_1 is detected or not. The rate of failures (to F_1) is however assumed to be the same for both D_{2u} and D_{2d}.

Using transition rates as in Figure 3, we can now easily write down the intensity matrix, Q, of this time continuous Markov chain. Now numbering the states as

OK = 1 $D_{1u} = 2$ $D_{1d} = 3$ $D_{2u} = 4$ $D_{2d} = 5$ $F_1 = 6$ OK* = 7

this 7x7 matrix is of the form

$$Q = \begin{bmatrix} A & K \\ 0 & 0 \end{bmatrix}$$

where A is a 5x5 matrix, and the "0"-s here are matrices consisting of zeros only.

Then using a suitable method for solving Markov chains, for example the computer package Maple, we can easily find the transition probabilities of the process. Let $X_n(t)$, n=1, 2, ….., be the state of the time continuous Markov chain at time t in n'th inspection interval, and let

$$p_{jk}(t) = \mathrm{P}(X_n(t) = k \mid X_n(0) = j)$$

(assumed independent of n). We denote this matrix of transition probabilities by $P(t)$ and get

$$P(t) = \begin{bmatrix} e^{tA} & A^{-1}(e^{tA} - I)K \\ 0 & I \end{bmatrix}$$

where I is the identity matrix of appropriate dimension.

2.3 Markov Chain for Transitions at Inspections

The states $X_n(0)$ and $X_n(T)$ are of particular interest, where T is the length of the interval. Now consider the transitions of state that may occur as the result of the inspection (occurring at times T, $2T$, ….). In order to be able to fit the model to the failure/inspection data, we now introduce the variables U_n and V_n. U_n tells the true state for a small piece of rail immediately before inspection, and V_n tells the true state immediately after inspection, i.e. at the start of the next inspection interval. Thus actually

$$U_n = X_n(T), \quad n = 1, 2, \ldots\ldots$$
$$V_n = X_{n+1}(0), \quad n = 1, 2, \ldots\ldots$$

Further, we introduce probabilities that degraded failures are detected by inspection:

q_1 = Probability that state D_1 of a line segment is detected.
q_2 = Probability that a degraded failure, D_2 is detected by the inspection; not knowing in advance that the state D_1 was reached.
q_3 = Probability that a degraded failure, D_2 is detected by the inspection; knowing in advance that the state D_1 was reached.

We can then introduce the matrix, R, for the transitions at the inspections, i.e. transitions from U_n to V_n.

$$\begin{array}{c} \quad\quad \begin{array}{ccccccc} OK & D_{1u} & D_{1d} & D_{2u} & D_{2d} & F_1 & OK^* \end{array} \\ R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-q_1 & q_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ q_2 & 0 & 0 & 1-q_2 & 0 & 0 & 0 \\ q_3 & 0 & 0 & 0 & 1-q_3 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The transition matrix for U_n equals R · P(T), and similarly the transition matrix for V_n equals P(T) · R; (also see [3]).

Now introducing the asymptotic distributions of U_n and V_n ,

$$\pi_k = \mathrm{P}(U_n = k); \quad k = 1, \ldots.., 7$$
$$\psi_k = \mathrm{P}(V_n = k); \quad k = 1, \ldots.., 7,$$

and the corresponding row vectors (vectors being bold)

$\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots\ldots, \pi_7)$
$\boldsymbol{\psi} = (\psi_1, \psi_2, \ldots\ldots, \psi_7)$

we have the two relations

$$\boldsymbol{\psi} = \boldsymbol{\pi} \cdot R$$
$$\boldsymbol{\pi} = \boldsymbol{\psi} \cdot P(T)$$

Thus the vector $\boldsymbol{\pi}$ is found from

$$\boldsymbol{\pi} = \boldsymbol{\pi} \cdot R \cdot P(T)$$

2.4 Overall Model and Assumptions

As indicated above the two Markov chains can be combined into one total model, see Figure 4. Here we give the state numbers 1, …., 7 in addition to the notation OK etc., and again for simplicity we ignore the state F_2. The solid lines represent the possible transitions within an inspection interval *T*, (cf. matrix P(*T*)). Recall that we make the simplifying assumption that one small piece of rail can only have one visit to OK* and F_1 (and F_2) within one inspection interval. Therefore we actually treat these as absorbing states in the time continuous Markov chain, and the process always start in state OK at the beginning of the next interval.

The dotted lines of Figure 4 indicate transitions at the end of the test interval (cf. matrix R). With probability q_1 we leave D_{1u} and with probability q_2 we leave D_{2u} (thus observing state D_2 but then going directly to OK which is the starting state at the beginning of next interval). Further, with probability q_3 we leave D_{2d} (thus actually observing D_2), and there will also be transitions from the "absorbing states" OK* and F_1 to OK.

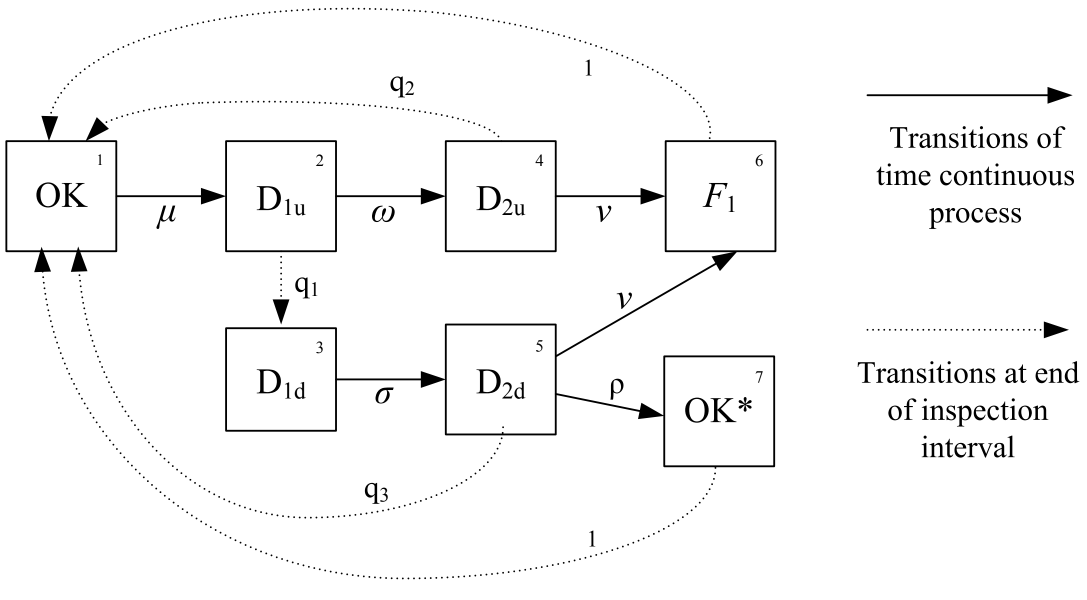

Figure 4: Overall failure/maintenance model, (state F_2 not included)

So one of the interesting aspects of the model is that we have a time-continuous Markov chain in the time spans between inspections, while at the inspections we have transitions following a time discrete Markov chain.

The model and the estimation of the parameters require some assumptions, e.g.:

- We use the asymptotic distributions of the processes U_n and V_n, thus these are assumed to be stationary processes; (the actual railway line is quite old so this is a rather realistic assumption).
- The probability distributions of the time continuous processes are identical for all inspection intervals (i.e. stationarity is assumed also in this respect).
- Implicitly we assume failures are equally distributed over the railway line (homogeneity). However, the estimated rates can be seen as averages for the line in question.
- We can treat the critically failed states as absorbing states that are also repaired at the inspections. This implies that one small piece of rail can not fail critically twice inside one inspection interval.
- Mean Time To Repair (MTTR) = 0.

In order to carry out the estimation of the unknown rates it is also required to estimate some parameters by expert judgments (operational experience); these are:

- The probabilities q_1, q_2 and q_3
- The rate (ρ) of reaching state D_2 under additional inspections when state D_1 is detected.

3. Estimation of Parameters

We here present the failure/inspection data for the *Dovrebanen* and estimate basic reliability parameters.

3.1 Input Data

The input parameters for the estimation of model parameters are listed in Table 1. Degraded failures have been recorded since 1st January 1991 and have been exposed to 8 tests by October 2002. Critical failures have been recorded since 1st January 1989, and assuming the same length of the test interval (T=4299/8=537 days) also for these, this implies that these have been exposed to 9.4 tests.

In total 800 degraded failures were observed. For 22 of these the severity was not recorded (i.e. not categorized as D_1 or D_2), and these 22 failures were just distributed proportionally amongst the two categories, giving in total 331 failures of type D_1 and 469 of type D_2.

Further, 20 of the D_2 failures have already been observed in state D_1 (i.e. coming from state D_{1d} and are actually monitored closely). It is then assumed that the test detects the transition to D_2, and so these failures are corrected, and there is a transition back to state OK. The 449 detected failures of type D_{2u} represent transitions from D_{1u}, i.e. the degradations are observed for the first time. These are a fraction q_2 of the actual number of rail pieces in state D_{2u}, and will by the test be brought back to the state OK. The other will remain in state D_{2u}.

Finally, the number of critical failures of type F_1 and F_2 are given as 249 and 81, respectively.

Table 1: Inputs to parameter estimation

Parameter definition	Parameter	Value
Length of rail	L	365 km = 365 000 m
Number of tests/inspections 1989-2002	n_{TF}	9.4
Number of tests/inspections 1991-2002	n_{TD}	8
Number of days, 1989-2002	N_1	5027
Number of days, 1991-2002	N_2	4299
Length of test/inspection interval	T	537 days
Number of observations in state D_1, (i.e. transitions from D_{1u} to D_{1d})	N_{D1}	331
Number of observations in D_2 when it was not known that state was degraded, (i.e. transitions from D_{2u})	N_{D2a}	449
Number of observations in D_2 when it was known that state was degraded, (i.e. transitions from D_{1d})	N_{D2b}	20
Number of observations in F_1	N_{F1}	249
Number of observations in F_2	N_{F2}	81
Probability of detecting D_1 failure at test	q_1	0.4 (expert judgement)
Probability of detecting D_2 failure at test (state D_1 not detected previously)	q_2	0.7 (expert judgement)
Probability of detecting D_2 failure at test (state D_1 already detected)	q_3	0.9 (expert judgement)
Rate of detecting D_2 in additional inspections; (assuming on the average two additional inspections within each interval T)	ρ	$(2/T)q_3$ (exp. judgem.)

3.2 Asymptotic Distributions and Transition Rates

It is now quite easy to estimate the distribution of U_n under stationarity. For instance, the estimate of π_6 is given by the number of observations in F_1. The equations for π_4 and π_2 are obtained similarly. Finally, the estimated π_5 and π_7 are found from the number of detections of state D_2 given that degradation D_1 is already known. One third of these observations are assumed to be carried out at the ordinary inspections (see bottom of Table 1), and two thirds at the additional inspections, thus resulting in transitions to state OK*. Finally we have the normalization equation, in total giving (for 1m of rail):

$$\hat{\pi}_2 \cdot q_1 = \frac{N_{D1}}{L \cdot n_{TD}} = 1.13 \cdot 10^{-4}$$

$$\hat{\pi}_4 \cdot q_2 = \frac{N_{D2a}}{L \cdot n_{TD}} = 1.54 \cdot 10^{-4}$$

$$\hat{\pi}_5 \cdot q_3 = \frac{1}{3} \cdot \frac{N_{D2b}}{L \cdot n_{TD}} = 2.28 \cdot 10^{-6}$$

$$\hat{\pi}_6 = \frac{N_{F1}}{L \cdot n_{TF}} = 7.30 \cdot 10^{-5}$$

$$\hat{\pi}_7 = \frac{2}{3} \cdot \frac{N_{D2b}}{L \cdot n_{TD}} = 4.57 \cdot 10^{-6}$$

$$\hat{\pi}_1 + \hat{\pi}_2 + \ldots\ldots + \hat{\pi}_6 = 1$$

Further, the estimate of π_3 can be obtained from the following relation:

$$\pi_3 = (\pi_2 \cdot q_1 + \pi_{3)} \cdot (1 - e^{-\sigma T})$$

The argument is that if $U_n = 3$, then either $U_{n-1} = 3$ *or* $U_{n-1} = 2$, and D_1 being detected, giving a transition from state 2 to 3, *and* in addition no transition has occurred in the last inspection interval. Rearranging this we get

$$\pi_3 = \pi_2 \cdot (e^{\sigma T} - 1) \cdot q_1$$

Now, a joint estimation of the stationary distribution, $\boldsymbol{\pi}$**,** and the unknown rates (μ, ω, σ, ν), are obtained by the following recursive approach:

- Starting with a value of σ, denoted σ_i (initially i=1), we get an distribution $\boldsymbol{\pi}(i)$.
- We use this distribution and the relation $\boldsymbol{\pi} = \boldsymbol{\pi} \cdot R \cdot P(T)$ to determine estimates of the transitions rates (μ, ω, σ, ν); (in particular giving a new σ denoted σ_{i+1})
- Calculate a new $\boldsymbol{\pi}(i+1) = \boldsymbol{\pi}(i) \cdot Re^{TQ}$, and use this to estimate μ, ω, σ, ν again

It was seen that this converges very rapidly (about 3-4 iterations), and using $q_1 = 0.4$, $q_2 = 0.7$ and $q_3 = 0.9$, $\rho = (2/T) \cdot q_3$ we get the following estimates for the stationary probabilities π_j (for 1 m rail):

$$\hat{\pi}_1 = 0.99935$$
$$\hat{\pi}_2 = 2.83 \cdot 10^{-4}$$
$$\hat{\pi}_3 = 7.25 \cdot 10^{-5}$$
$$\hat{\pi}_4 = 2.20 \cdot 10^{-4}$$
$$\hat{\pi}_5 = 2.54 \cdot 10^{-6}$$
$$\hat{\pi}_6 = 7.30 \cdot 10^{-5}$$
$$\hat{\pi}_7 = 4.57 \cdot 10^{-6}$$

Further we get the following estimated rates (per day):

$$\hat{\mu} = 5.9 \cdot 10^{-7} \text{ /m}$$
$$\hat{\omega} = 1.6 \cdot 10^{-3}$$
$$\hat{\sigma} = 9.2 \cdot 10^{-4}$$
$$\hat{\nu} = 9.3 \cdot 10^{-4}$$
$$\hat{\rho} = 3.4 \cdot 10^{-3}$$
$$\hat{\lambda} = 4.4 \cdot 10^{-8} \text{ /m}$$

Observe that the estimate for ρ was obtained directly from expert judgment (Table 1), and λ was estimated directly from the number of F_2 failures, (N_{F2}) the total number of days N_1, and the rail length, i.e. $\hat{\lambda} = N_{F2}/(N_1 \cdot L)$.

Further, observe that now the distribution of V_n is also found using the R-matrix. There are five possible states at the start of a test interval, and the estimates are found from the following relations

$$\Psi_1 = P(V_n = 1) = \pi_1 + \pi_4 \cdot q_2 + \pi_5 \cdot q_3 + \pi_6 + \pi_7$$
$$\Psi_2 = P(V_n = 2) = \pi_2 \cdot (1 - q_1)$$
$$\Psi_3 = P(V_n = 3) = \pi_2 \cdot q_1 + \pi_3$$
$$\Psi_4 = P(V_n = 4) = \pi_4 \cdot (1 - q_2)$$
$$\Psi_5 = P(V_n = 5) = \pi_5 \cdot (1 - q_3)$$

giving

$$(\hat{\psi}_1, \hat{\psi}_2, \ldots, \hat{\psi}_5) = (0.99958, 1.7\cdot10^{-4}, 1.9\cdot10^{-4}, 6.6\cdot10^{-5}, 2.5\cdot10^{-7}).$$

3.3 Additional Results

Now having estimated the transition rates, some basic results concerning sojourn times directly follow. We use the notation E[*T, i*] for the mean sojourn times in state *i*. Given no maintenance, we have a pure continuous process and these mean times are given as the inverse of corresponding rates, see Table 2. This table also gives $MTTF_i$ Mean Time To Failure for failure mode *i* (i.e. F_1 and F_2). Also the overall MTTF is given. Note that these results corresponds to so-called *naked* failure rates (no repair is performed), see e.g. [8, 9, 10]. The rate 1/MTTF = ($1/MTTF_1$ + $1/MTTF_2$) could also be referred to as the (asymptotic) Rate of Occurrence of Failure, ROCOF, (see [11]) when there is no maintenance.

Table 2: The estimated sojourn times and MTTF, assuming no maintenance (T=∞). For a piece of rail of length 1 km.

Parameter	Estimate
E[T, OK] $(1/\mu)$	4.6 years
E[T, D_{1u}] $(1/\omega)$	1.7 years
E[T, D_{2u}] $(1/\nu)$	2.9 years
$MTTF_1$ $(1/\mu+1/\omega+1/\nu)$	9.2 years
$MTTF_2$ $(1/\lambda)$	62 years
MTTF $(1/MTTF_1 + 1/MTTF_2)^{-1}$	8.0 years

4. Maintenance Optimization

Now having established the estimates of the rates λ, μ, ω, σ and ν it is of interest to consider the effect of various levels of maintenance on basic reliability parameters like:

- Frequency (rate) of entries into failure states
- MTTF

There are two parameters that we can control:

- Length of inspection interval, *T*
- Frequency of additional inspections, (cf. ρ), when degradation in state D_1 is detected

As the number of entries to the critically failed state F_2 does not change with the maintenance level, we keep focusing on the failures due to degradation. In particular the asymptotic rate of entering F_1 (i.e. $ROCOF_1$), and its inverse ($MTTF_1$) for various values of T is most interesting.

To find such entry rates we need the over-all average probability of the time continuous process being in various states. The probability for the process $X(t)$ to be in state k at time t equals; (remember that $\psi_5 = \psi_6 = 0$):

$$p_k(t) = \sum_{j=1}^{5} \psi_j \, p_{jk}(t)$$

Here $p_{jk}(t)$ are the elements of the transition matrix $P(t)$. Now the "average probability" to be in state k equals

$$p_k^* = \sum_{j=1}^{5} \psi_j \, p_{jk}^*$$

where

$$p_{jk}^* = \frac{1}{T}\int_0^T p_{jk}(t)\, dt$$

First, the "overall" (average) probability of the time continuous process to be in state OK is found as (see Figure 3)

$$\text{P(OK)} = \psi_1 \cdot \frac{1}{T} \cdot \int_0^T p_{11}(t)\, dt$$

Since obviously $p_{11}(t) = e^{-\mu t}$.

$$\text{P(OK)} = \psi_1 \cdot [1- e^{-\mu T}] / (\mu \cdot T) \approx [1- e^{-\mu T}] / (\mu \cdot T)$$

The rate of entries into degradation state D_1 (actually D_{1u}) is found as

$$\text{ROCOD}_{1u} = \text{P(OK)} \cdot \mu \approx [1- e^{-\mu T}] / T$$

Similarly, the probability to be in state D_{1u} equals

$$\text{P}(D_{1u}) = \psi_1 \cdot p_{12}^* + \psi_2 \cdot p_{22}^*$$

Here

$$p_{12}(t) = [\mu \cdot/(\mu-\omega)] \cdot [e^{-\omega t} - e^{-\mu t}]$$

$$p_{22}(t) = e^{-\omega t}$$

Thus, it will follow

$$\text{P}(D_{1u}) = \psi_1 \cdot [\mu \cdot/(\mu-\omega)] \cdot [(1-e^{-\omega T})/\omega - (1-e^{-\mu T})/\mu] / T + \psi_2 \cdot [1- e^{-\omega T}]/T$$

The rate into D_{2u} equals

$ROCOD_{2u} = P(D_{1u})\cdot\omega$

The most interesting part is obviously the rate of entries into the critical failure state F_1. We then need the probabilities to be in states D_{2u} or D_{2d}. Now

$P(D_{2u}) = \psi_1 \cdot p_{14}{}^* + \psi_2 \cdot p_{24}{}^* + \psi_4 \cdot p_{44}{}^*$

$P(D_{2d}) = \psi_3 \cdot p_{35}{}^* + \psi_5 \cdot p_{55}{}^*$

which can be computed in the same way as above. Now the asymptotic rate into F_1

$ROCOF_1 = [P(D_{2u}) + P(D_{2d})] \cdot v$

so for the maintained system, the asymptotic $MTTF_1 = 1/ ROCOF_1$ is given by the above formulas.

We computed $ROCOD_{1u}$, $ROCOD_{2u}$, $ROCOF_1$ and $MTTF_1$ for a few values of T, and also for some alternative values of ρ. Table 3 gives the results for T=365 days and 730 days together with the actual value of the observations (T=537 days). The mean number of observations in state F_1 for a duration corresponding to the actual data (5027 days) are also given.

Table 3: Reliability parameters for various values of *T*

Parameter	Inspection interval, T		
	365 days	537 days	730 days
$ROCOD_{1u}$ (per day and km)	$5.88 \cdot 10^{-4}$	$5.88 \cdot 10^{-4}$	$5.88 \cdot 10^{-4}$
$ROCOD_{2u}$ (per day and km)	$2.98 \cdot 10^{-4}$	$3.50 \cdot 10^{-4}$	$3.88 \cdot 10^{-4}$
$ROCOF_1$ (per day and km)	$0.98 \cdot 10^{-4}$	$1.42 \cdot 10^{-4}$	$1.85 \cdot 10^{-4}$
$MTTF_1$ for 1 km rail (years)	28.0	19.3	14.8
Expected no. of failures (in 5027 days)	180	261	339

In Table 4 we use T = 537 days, but vary the rate ρ. The nominal is the value used in the analyses above and ρ is given as percentage of this.

Table 4: Reliability parameters for various values of *ρ*

ρ (rate of increased inspection by detecting state D_1)	$ROCOF_1$ (per day and km)	$MTTF_1$ for 1 km rail (years)	Exp. no. of failures (in 5027 days)
25%	$1.50 \cdot 10^{-4}$	18.3	276
50%	$1.47 \cdot 10^{-4}$	18.6	270
100%	$1.42 \cdot 10^{-4}$	19.3	261
200%	$1.36 \cdot 10^{-4}$	20.1	250
400%	$1.31 \cdot 10^{-4}$	20.9	240

5. Conclusions

The paper reports a realistic real life case study. The situation is rather complex, leading to a model where we combine a model for failure development and a model for maintenance. The former describes a degradation mechanism through various states (phase type failure distribution), while the latter describes a maintenance strategy being a kind of mixture of preventive maintenance and condition monitoring.

Our approach uses a combination of a time continuous and a time discrete Markov chain. The model is made as simple as possible in order to achieve transparency and simple interpretation of the results. This has been achieved, for example, by reducing the number of degradation states.

The approach is utilizing a combination of "hard" and "soft" input, which is often the case in a practical situation. The soft input consists of expert judgment based on operational experience to assess probabilities to detect degradation states by inspection, and frequency of additional inspections after this is detected using "best estimates". The hard input is given by the actual observations of degradations for a rather long period of operation.

The estimated model shows for instance that the MTTF for critical degradation failures of 1 km length of rail decreases from 28.0 years when inspection interval is 1 year to 14.8 years when the inspection interval is 2 years. Similarly, mean number of failures is doubled when T increases from 1 year to 2 years. The frequency of increased inspections when a degraded failure is detected has less effect on the results.

The computed estimates are valid for the given line only: for another line with a different state of the line and the environment one would obviously get different estimates for the transition rates. The approach is, however, generally applicable. The paper is demonstrating the usefulness of analytic models to optimize maintenance. Even simple, basic models like Markov chains may lead to significant achievement with respect to optimal use of resources.

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